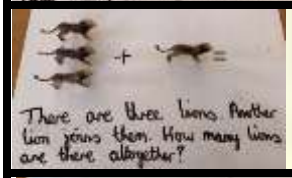
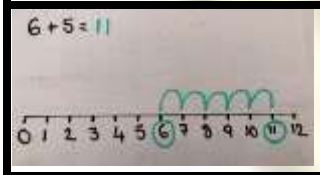
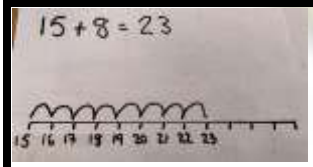
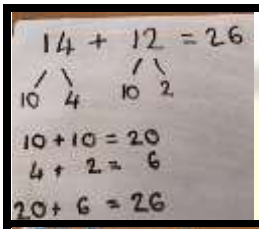
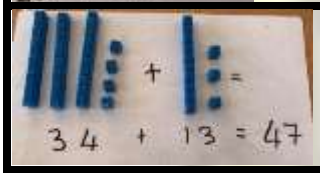
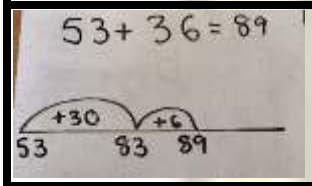
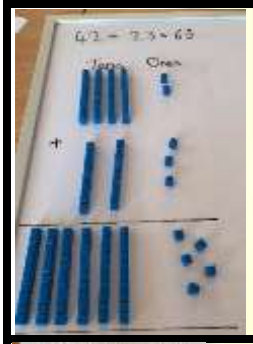
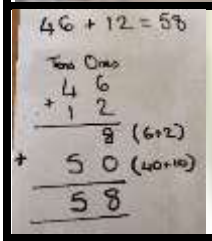
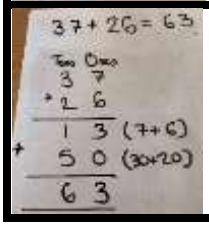
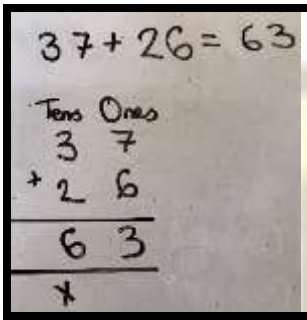
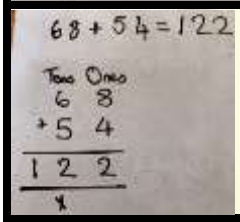
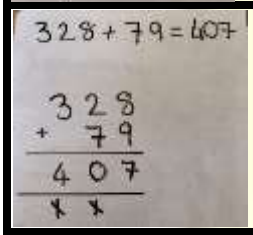
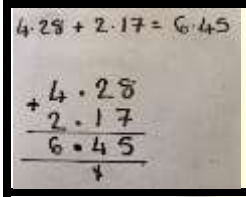
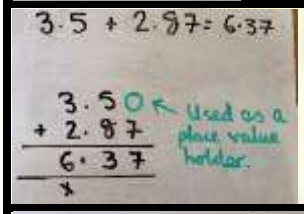
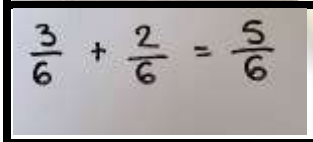
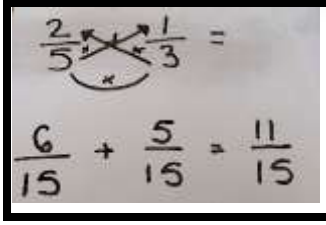
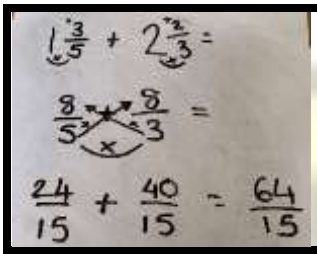
Examples of the calculation process. This shows the development of the approach to calculating the four operations. These should be used flexibly to suit the needs of the learners, there is no requirement for every child to complete every step in this guide.

Addition

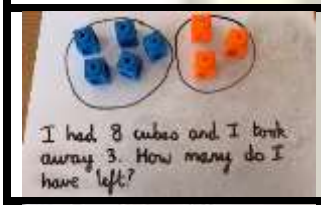
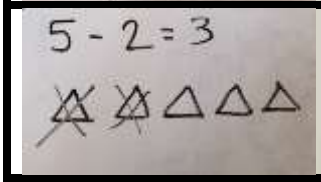
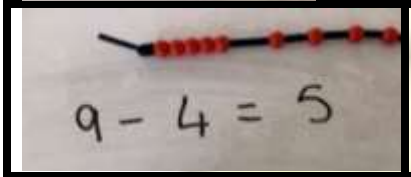
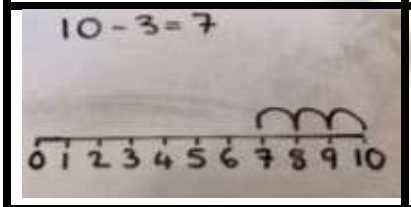
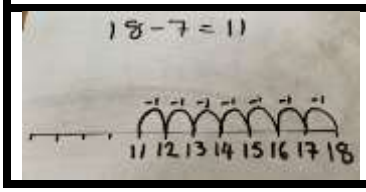
Example of method	Comments
	The children will start by counting objects and being asked to state how many there are. Counting physical objects that they can pick up and manipulate is vital to developing conceptual understanding.
	This is then extended to asking adding more objects to the group and asking the children to count how many there are now. This introduction to addition makes it clear to the children what addition means.
	Number sentences can then be created using the concrete objects, this introduced the addition symbol and the equals symbol to the children.
	The use of a variety of manipulatives, for example bead strings, can help the children to work with number in a way that is comfortable for them. In this example, the children can physically 'add' the beads to the pile to get the new total.
	When beginning with number lines, it is important for the children to see a number line like this, where all numbers are visible, as this helps them to visualise the numbers in the sequence. Individual jumps at this stage are recommended, so that the children can confidently count on.
	The use of an empty number line is a vital skill for developing addition strategies, especially as the numbers begin to get larger and it becomes impractical to create a number line starting from zero. Initially, the children should continue with individual jumps of one to build confidence.
	Using a hundred square as an alternative to a number line may be more beneficial for some children, as it shows the number order. When using this approach with the children, ensure that the crossing of the tens boundary is explained clearly, to avoid confusion.

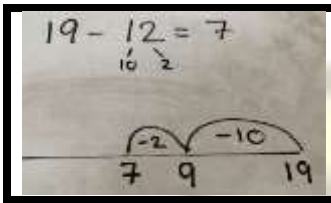
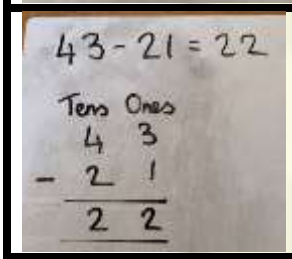
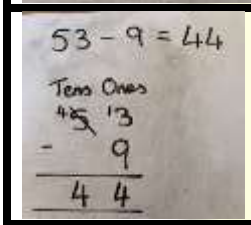
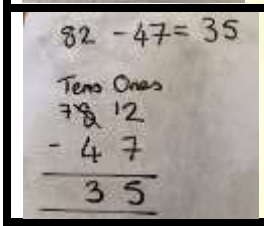
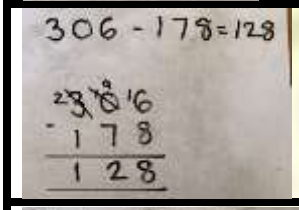
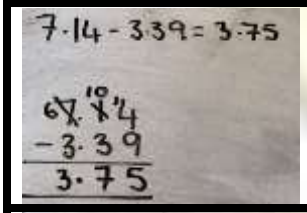
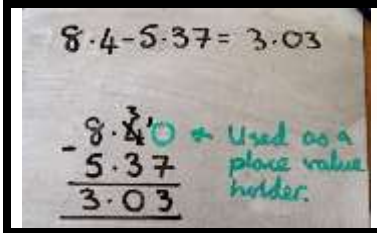
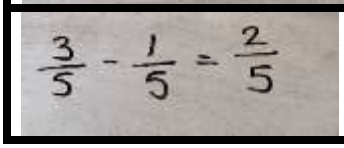
	As the children become exposed to larger numbers, a vital skill in their ability to manipulate number problems is partitioning. This initially should be through partitioning just into the place value amounts (tens and ones shown here). As they become more confident, they can partition the numbers in different ways, in situations where it would make their calculations more efficient.
	The use of the base 10 resource becomes more important with using these numbers as children begin the movement from less formal to more formal methods. It also allows the children to model larger questions in a concrete way, as well as allowing them to see the difference between the place value columns.
	The children should then be able to apply their partitioning knowledge to the empty number line model they have already experienced. This will allow them to become more efficient with adding using a number line.
	There should also by now be an introduction to the column method for addition, if the children are ready for this. To start with, this should be modelled with resources, like the base 10 shown here, so that the children understand the value of the different place value columns. When introducing this method, ensure that the children do not have questions that cross the tens boundary and require carrying over, until they become confident with the method.
	The next step for the children would be to see a written version of the modelled question. To start with, showing the expanded column method is helpful to show the children the process of adding in this way and the order in which you complete the questions. Labelling the columns helps to reinforce the value of the columns for the children.
	The introduction of questions that cross the tens boundary using the expanded column method allows the children to see why there is a number to carry over. This method should be used until the children are confident, before moving onto the formal method.

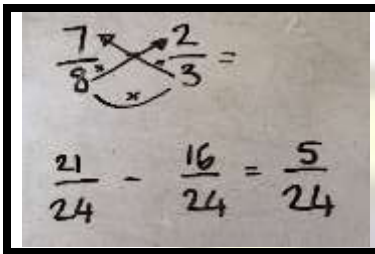
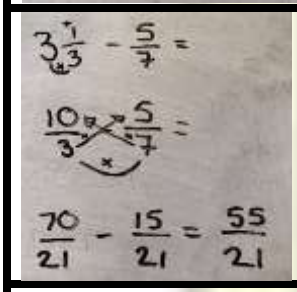
	After becoming confident with the expanded method, the children should move onto the formal method of column addition. This becomes the method the children require for any questions larger than the one shown, when using the expanded method would become impractical to complete. Ensure that numbers carried over are taught consistently, shown here under the question, and are crossed out neatly once they've been added to the total. This avoids confusion with other calculation methods.
	The children should then be shown addition problems that cross the next place value boundary, which was not in the original question. This allows the children to reinforce place value knowledge as well as knowing what to do in a situation where it appears that they 'don't have a column' to place the number into.
	The next example that the children should be exposed to is questions where the numbers have a different amount of digits and the children are required to correctly line up the numbers by place value columns. It may be beneficial here to return to the labelling of the place value columns to reinforce this knowledge.
	When adding numbers with decimal points in, ensure that the children are aware that the decimal points must be lined up to correctly answer the question. The method for addition doesn't change, but the decimal points need to be in line and also in the answer before they start calculating.
	Questions like this often cause problems for children with lining up the questions correctly. Encourage the children to put the decimal points in first and fill any empty spaces with a place holder zero. This is required in subtraction questions, but is also helpful for showing the children that a number does exist in that column, even if it is blank.
	When adding fractions with the same denominator (bottom number), you can just add the numerators (top numbers) together. The denominator stays the same in the answer.
	When factions have different denominators, they have to be converted to a common denominator before the calculation can take place. This method shown is a simple method for ensuring the fractions remain equivalent, but also have a common denominator. Once the fractions are converted, they can be added in the same way as shown above.

	When calculating with mixed numbers, it is best to teach the children to convert the mixed number to an improper fraction before beginning the calculation, as shown here. They can then follow the process of finding common denominators as previously explained, to get to a solution. The children can be shown how to convert an improper fraction back into a mixed number, but the children will not be penalised if their answer is left as an improper fraction.

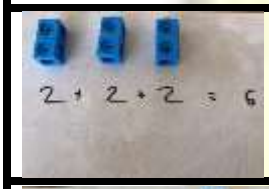
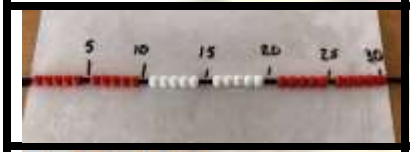
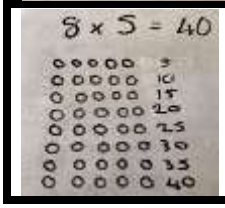
Subtraction

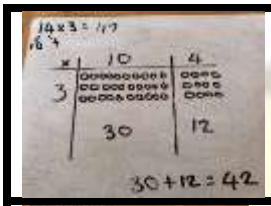
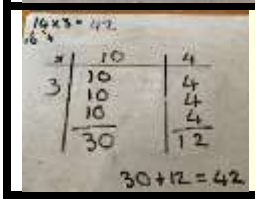
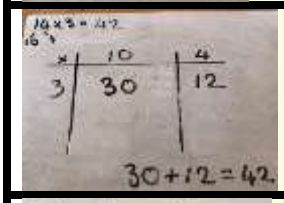
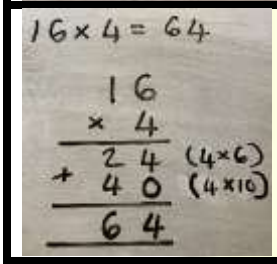
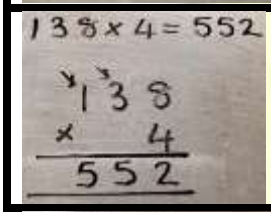
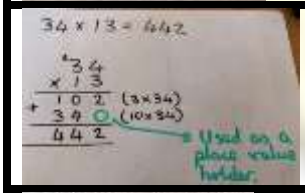
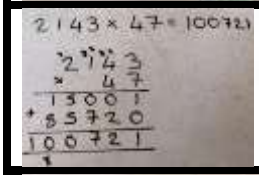
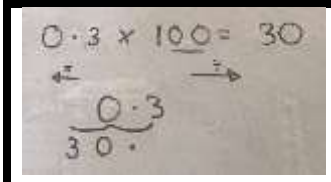
Example of method	Comments
	The children should begin their experiences of subtracting with the use of manipulatives and being asked when happens if you 'take away' some of the items. This introduces the idea that when subtracting, the total becomes smaller.
	The children can visually represent the subtraction questions by drawing images of the numbers and crossing out the ones they are subtracting. This allows them to see what they are doing with the calculation.
	The use of a variety of manipulatives, for example bead strings, can help the children to work with number in a way that is comfortable for them. In this example, the children can physically 'subtract' the beads from the pile to get the new total.
	Using a hundred square as an alternative to a number line may be more beneficial for some children, as it shows the number order. When using this approach with the children, ensure that the crossing of the tens boundary is explained clearly, to avoid confusion.
	When subtracting with number lines, the children are taught to start at the larger number and count back. It is important for the children to see a number line like this, where all numbers are visible, as this helps them to visualise the numbers in the sequence. Individual jumps at this stage are recommended, so that the children can confidently count back.
	The use of an empty number line is a vital skill for developing subtraction strategies, especially as the numbers begin to get larger and it becomes impractical to create a number line starting from zero. Initially, the children should continue with individual jumps of one to build confidence.

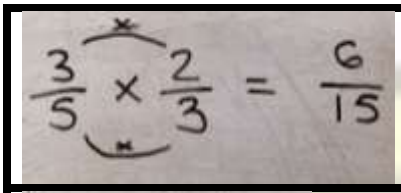
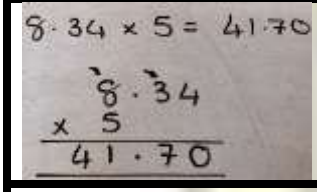
	The children should then be able to apply their partitioning knowledge to the empty number line model they have already experienced. This will allow them to become more efficient with subtracting using a number line and subtracting larger jumps at once.
	The children should be introduced to the column method for subtraction through the use of base 10, moving onto the written method shown here. There should be no exchanging and regrouping initially, as this is a step to be introduced once the children are confident in the method and starting the subtraction by subtracting from the smallest column.
	Exchanging and regrouping should be introduced slowly, and starting with only one exchange to make. It is vital to get terminology correct here, as the use of 'borrowing' will confuse children, as it implies giving back, which does not happen in this method.
	The progression is to make the numbers larger so that there is a subtraction to do in each column, with some exchanging to take place.
	Once the children are more confident with exchanging, they should be exposed to questions where multiple exchanging is required, and exchanging across more than one column.
	When subtracting numbers with decimal points in, ensure that the children are aware that the decimal points must be lined up to correctly answer the question. The method for subtraction doesn't change, but the decimal points need to be in line and also in the answer before they start calculating.
	Questions like this often cause problems for children with lining up the questions correctly. Ensure the children to put the decimal points in first and fill any empty spaces with a place holder zero. This is required in subtraction questions, as it will mean that exchanging needs to take place in order for the children to correctly answer the question.
	When subtracting fractions with the same denominator (bottom number), you can just subtract the numerators (top numbers) together. The denominator stays the same in the answer.

	When factions have different denominators, they have to be converted to a common denominator before the calculation can take place. This method shown is a simple method for ensuring the fractions remain equivalent, but also have a common denominator. Once the fractions are converted, they can be subtracted in the same way as shown above. It is vital that the numbers are correctly place when converting because if they are not, the calculation will not be correct.
	When calculating with mixed numbers, it is best to teach the children to convert the mixed number to an improper fraction before beginning the calculation, as shown here. They can then follow the process of finding common denominators as previously explained, to get to a solution. The children can be shown how to convert an improper fraction back into a mixed number, but the children will not be penalised if their answer is left as an improper fraction.

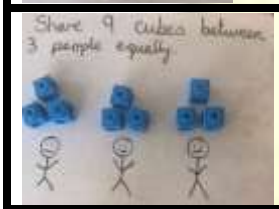
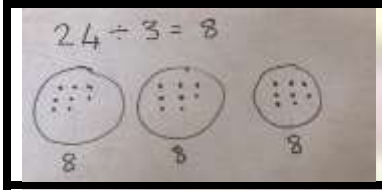
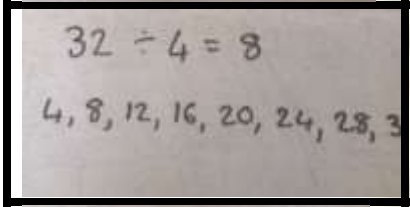
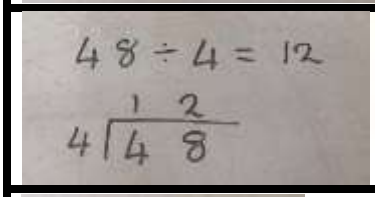
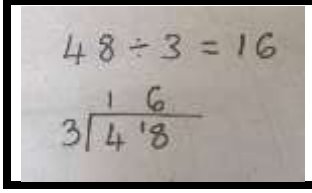
Multiplication

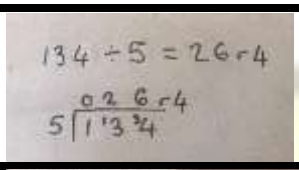
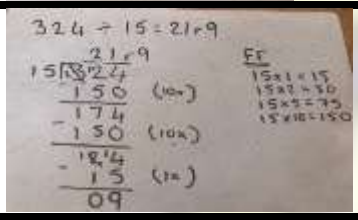
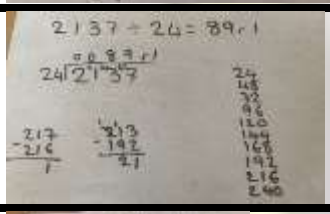
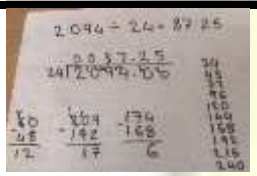
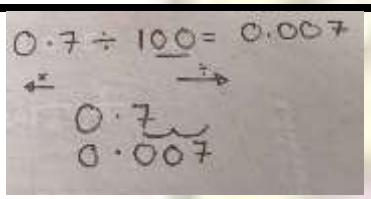
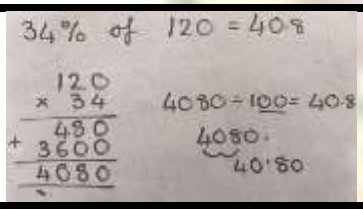
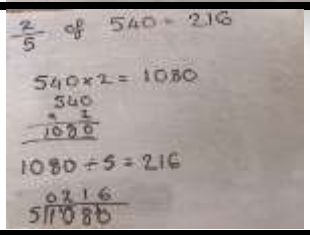
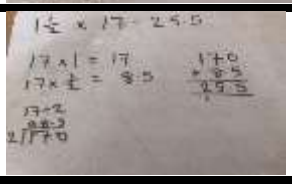
Example of method	Comments
	The children should begin their exposure to multiplication by grouping objects and finding 'lots of' an amount and totaling it up. This also highlights the relationship between addition and multiplication, as multiplication is repeated addition.
	The children should be taught doubling and halving early on, with doubling being a multiplication by two. This is easily represented here and gives the children a visual understanding of what doubling means.
	The children can represent the grouping aspect of multiplication through using a bead string, showing how the groups repeat to make the total.
	The children should then be shown arrays in a concrete form, as this also helps to visualise the process of multiplying. This should be accompanied by written recordings to show the process.
	Concrete arrays can be replaced by pictorial, hand drawn arrays once the children are more confident. Counting up and recording the multiples at the side of the array allow the children to continue to cement the link between repeated addition and multiplication.

	The children should then be introduced to the grid method of multiplication, for multiplying larger numbers. This can be easily transitioned by including arrays or concrete resources, such as base 10, inside the grid. This will make the link between arrays and the grid clearer.
	As the children become more confident, they may feel they do not need to draw out an array, so they could simplify it to the writing of the numbers as shown here. This intermediate step reinforces what is required in each part of the grid.
	The children should then become confident at using the grid method accurately and efficiently without the need for additional working out inside the grid. This method can continue to be used with larger numbers if the children are more confident with this.
	The children should be introduced to the formal method of column multiplication through the use of expanded column method, similar to the introduction to column addition. This should only be used to introduce the carrying aspect of multiplication in a column, as with larger numbers it becomes less efficient. Labelling the multiplications being completed allows the children to see the working more clearly.
	The children should then progress onto the formal method of column multiplication, with carrying over included. Carrying over the numbers above the question here is helpful, as the numbers do not then get mixed in with the answer line, particularly when multiplying by more than one digit.
	When multiplying by more than one digit, it is vital that the children include the place value holder zero on the second line of calculating. Encouraging the children to include the brackets for the calculating and writing in the zero first will help them remember this zero.
	The children will then progress through the multiplication process, up to calculating a 4-digit number multiplied by a 2-digit number. Care should be taken to always ensure that numbers are correctly lined up, so that confusion can be avoided.
	When multiplying by 10, 100 or 1000, using the column method is not the most efficient method. The children can move across place value columns as shown in this example. The children must understand that when we multiply by 10, 100 or 1000, **we do not 'add a zero'** and when multiplying with decimals, the decimal point **does not move**.

	When multiplying two fractions together, we simply multiply the numerators together and multiply the denominators together. If a question with a mixed number appears, convert it to an improper fraction before multiplying the fractions together.
	When multiplying decimal numbers, lining up the numbers is important to maintain place value knowledge. However, place value holders are not required in multiplying, we just multiply the digit we have.

Division

Example of method	Comments
	The children should begin their exposure to division by sharing objects into equal groups, and then verbalising how many equal groups they have. This helps the children to understand the concept of division.
	The children should then experience questions where they share an amount between people, giving real life context to the questions that they are given. They should continue to model with concrete resources at this stage.
	The children should then progress onto a more pictorial approach to division, sharing by drawing arrays in the groups. This helps them to develop efficiency with dividing larger numbers, whilst still maintaining the sharing approach.
	The children may find that writing out the multiples of the number they are dividing by useful in making the transition from 'sharing' to 'grouping', which is how they divide using formal methods. This also makes the link between multiplication and division clearer for the children.
	When introducing the formal method of division to the children, they should begin by only using numbers where no remainders are required to be carried over, until the children are more confident with the method.
	The children should then progress onto questions where a remainder is required to be carried over to the next number. The questions at this stage should divide equally, as the children are still developing confidence with this method.

	The children should begin working with remainders when dividing by a single digit number. They should be recording the remainder as only a remainder, as shown here.
	As the numbers become larger, the children can be introduced to the 'chunking' method of division. This is an introduction to long division, and requires the children to use 'friendly facts' to aid them in their division. This also reinforces the link between multiplication and division and helps the children to see how we 'group' a number when dividing.
	When using short division with larger numbers, it may be beneficial for the children to write out the multiples of the divisor (the number they are dividing by). This will help them to maintain accuracy with handling larger numbers, without needing to keep all the numbers mentally.
	The children should then also be shown how to express a remainder as a decimal, as shown here. This method then gives the children an option to either leave an answer as a remainder or a decimal, should a question require a specific method.
	When dividing by 10, 100 or 1000, using the formal method is not the most efficient method. The children can move across place value columns as shown in this example. The children must understand that when we divide by 10, 100 or 1000, **we do not 'take away a zero'** and when dividing with decimals, the decimal point **does not move**.
	Finding a percentage of an amount requires the combination of multiplication and division skills and can be completed in the method shown here. The children should ensure their answer is appropriate, and less than the number in the question if finding a percentage less than 100%.
	Finding a fraction of an amount requires the combination of multiplication and division skills and can be completed in the method shown here. By multiplying by the numerator first, you ensure that both numbers for the second stage of the calculation are whole numbers, which then makes the division by the denominator simpler for the children to handle.
	Multiplying an amount by a mixed number can be completed using this method. This again combines knowledge of multiplication and divisions and requires more than one step.